# Betting and Basic Probability

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## Sports Betting

Suppose you are betting that the Rams will beat the Patriots in the Super Bowl. Suppose the Rams win with probability $p$. Suppose you bet \$$X$ to win \$$Y$ (for instance, you bet \$100 to profit \$130). Then your expected profit is

$$\mathbb{E}[\text{profit}] = pY + (1-p)(-X)$$

because with probability $p$ you profit \$$Y$ and with probability $1-p$ you lose \$$X$. You would like to have a positive expected profit, because this indicates that in the long run you are expected to profit from making the bet over and over. Now, we see that

$$\mathbb{E}[\text{profit}] = pY + (1-p)(-X) > 0$$

if and only if

$$\boxed{p > \frac{X}{X+Y}}$$

We call the term $\frac{X}{X+Y}$ the *pot odds*, and come to the conclusion that it is a good idea to make the bet in the long run if the above inequality holds (i.e., if the probability of winning the bet exceeds the pot odds), and a bad idea to make the bet if it doesn't hold.

The main problem with this strategy is that it is difficult to obtain the probability $p$ that the Rams win, if it even exists. We may resort to statistical methods (or to our intuition) to estimate $p$, but to obtain $p$ with certainty is impossible. But if for some reason you feel deeply that you have obtained a value for $p$, I encourage you to consider this strategy.

In poker, however, there are many instances in which we can find $p$.

## Poker

We call $\frac{X}{X+Y}$ the pot odds because we can apply this strategy to poker. Suppose you are playing poker and you have to call \$$X$ to stay in the hand, and the total pot size is \$$Y$. So, you have to call \$$X$ to profit \$$Y$ if you win. Then we find ourselves in a similar situation as the Super Bowl example - you bet \$$X$ to profit \$$Y$. Now, if you know the probability $p$ that you win the hand, then you can employ the previous strategy to ensure profit in the long run.

Fortunately, in poker there are instances where we can easily determine $p$. For instance, suppose you are playing Texas Holdem and have an A♡7♡ of Hearts in your hand, and the table cards are Q♡6♠3♡2♣. So, only the river card (one more table card) remains. Since you have two hearts and the Ace of hearts in your hand, a flush (the nut-flush) virtually guarantees that you win the hand. Now, there are 13 hearts in the deck and 4 have been dealt, so 9 remain in the deck. We call the cards we need to win (these 9 hearts) *outs*. You have seen 6 cards (2 in your hand, 4 on the table), so 46 cards remain unkown to you, so the probability of drawing a heart on the last card is 9/46. Since 46 is so close to 50, at the poker table we can approximate this probability by calling it 9/50, and doubling it gives us 18/100, which gives us our probability of winning on the river: $p = .18$. The fast way to obtain this .18 is by taking the number of outs (9) and doubling it (18). So, if we have to call \$$X$ with a pot size of \$$Y$, we should make the call if and only if

$$p > \frac{X}{X+Y}$$

We can perform similar calculations to find $p$ in many other instances. As we saw in the example, if only the river card remains undrawn,

$$p = \frac{2 * \binom{\text{number}}{\text{of outs}}}{100}$$

## Poker Strategy, Succintly

If you can determine the probability that you win a hand and need to call a certain amount to stay in the hand, make the call if

$$\boxed{\begin{matrix}\text{probability}\\ \text{you win}\end{matrix} > \frac{\text{call amount}}{\text{call amount} + \text{pot size}}}$$

and fold if not.