Lab: Kelly Betting

1. A Smart -EV Bet

Kelly (1956) extended his analysis to horses

- m horses $1, \dots, m$
- true (known) win probabilities $p_1, \dots, p_m$
- decimal odds $\alpha_1, \dots, \alpha_m$
- payroll fractions $f = (f_1, \dots, f_m)$ to bet on each horse.
- make N sequential bets, solve for f.

He used Lagrange Multipliers / KKT conditions to maximize expected log bankroll after N bets, yielding the Kelly allocations f via this algorithm:

(a) Permute indices so that $p(s)\alpha_s \geqq p(s+1)\alpha_{s+1}$

(b) Set b equal to the minimum positive value of

$$\frac{1-p_t}{1-\sigma_t} \quad \text{where} \quad p_t = \sum_1^t p(s), \;\; \sigma_t = \sum_1^t \frac{1}{\alpha_s}$$

(c) Set $f(s) = p(s) - b/\alpha_s$ or zero, whichever is larger. (The $f(s)$ will sum to $1 - b$.) if the odds are fair).

Consider a horse Race with $m=3$ horses.
Find an example of true horse win probabilities (P_1, P_2, P_3) and decimal odds $(\alpha_1, \alpha_2, \alpha_3)$ such that it is Kelly optimal to bet on a $-EV$ horse.
The Kelly bet allocations (f_1, f_2, f_3) according to the above algorithm must consist of $f_i > 0$ for some horse i, where betting on horse i is $-EV$: $P_i \alpha_i - 1 < 0$.

Then, $M=100$ times, simulate $N=1000$ bets on this horse Race and Record your bankRoll at the end using allocations (f_1, f_2, f_3) according to ① Kelly and ② separately letting $f_i = 0$ (where i is the $-EV$ horse). Which yields more wealth on average?

Why is it possible that making a $-EV$ bet is smart?

If the game is profitable on the whole, you Should put money into the game.
If you only bet on the $+EV$ horse you can lose more than if you hedge.