

# A Hierarchical Bayesian Model of Pitch Framing

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## Abstract

Abstract (PITCH), a hierarchical Bayesian model of pitch framing in music. The model is based on the idea that pitch framing is a process of inferring the underlying structure of a piece of music from the observed pitch contours. The model is trained on a large corpus of music and is able to predict the pitch contours of new pieces of music. The model is also able to generate pitch contours that are similar to those of the training data. The model is implemented in Python and is available as a package on PyPI.

## 1 Introduction

Abstract (PITCH), a hierarchical Bayesian model of pitch framing in music. The model is based on the idea that pitch framing is a process of inferring the underlying structure of a piece of music from the observed pitch contours. The model is trained on a large corpus of music and is able to predict the pitch contours of new pieces of music. The model is also able to generate pitch contours that are similar to those of the training data. The model is implemented in Python and is available as a package on PyPI.



Fig: [http://www.espn.com/mlb/story/\\_/id/111111](http://www.espn.com/mlb/story/_/id/111111)  
[http://www.espn.com/mlb/story/\\_/id/111111](http://www.espn.com/mlb/story/_/id/111111)  
[http://www.espn.com/mlb/story/\\_/id/111111](http://www.espn.com/mlb/story/_/id/111111)

Baseball pitch

strike

zone, [http://www.espn.com/mlb/story/\\_/id/111111](http://www.espn.com/mlb/story/_/id/111111)

[http://www.espn.com/mlb/story/\\_/id/111111](http://www.espn.com/mlb/story/_/id/111111)

Keuchel pitch called strike

[http://www.espn.com/mlb/story/\\_/id/111111](http://www.espn.com/mlb/story/_/id/111111)

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[http://www.espn.com/mlb/story/\\_/id/111111](http://www.espn.com/mlb/story/_/id/111111) (e.g. [Park, 2011](http://www.espn.com/mlb/story/_/id/111111); [F](http://www.espn.com/mlb/story/_/id/111111)

[Ch, 2015](http://www.espn.com/mlb/story/_/id/111111)), [Asano](http://www.espn.com/mlb/story/_/id/111111) (e.g. [Kim, Kim, 2014](http://www.espn.com/mlb/story/_/id/111111); [M](http://www.espn.com/mlb/story/_/id/111111)

[2014](http://www.espn.com/mlb/story/_/id/111111)), [Ch, 2016](http://www.espn.com/mlb/story/_/id/111111)). [D](http://www.espn.com/mlb/story/_/id/111111)

[Ch, 2015](http://www.espn.com/mlb/story/_/id/111111)

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[Ch, 2015](http://www.espn.com/mlb/story/_/id/111111) (e.g. [L, 2013](http://www.espn.com/mlb/story/_/id/111111); [P, 2014](http://www.espn.com/mlb/story/_/id/111111);

[F, 2015](http://www.espn.com/mlb/story/_/id/111111)) [Ch, 2015](http://www.espn.com/mlb/story/_/id/111111) (e.g. [D, 2014](http://www.espn.com/mlb/story/_/id/111111); [H, 2014](http://www.espn.com/mlb/story/_/id/111111)) [Ch, 2015](http://www.espn.com/mlb/story/_/id/111111)



linear regression

linear regression (regression).

linear regression

linear regression Model (2015)

linear regression

linear regression

linear

linear regression

linear regression

linear regression

linear regression Online

linear regression

linear regression (2011) fit

linear regression

linear regression

linear regression (2008)'s

linear regression (2015).

linear regression fit

linear regression

linear regression

linear regression

linear

linear regression

linear regression

linear regression

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linear regression (2015)'s linear regression (2011)'s

linear regression

linear regression

definition

Binary Choice

$$y = 1 \text{ if}$$

and

$$y = 0 \text{ if}$$

b, ca, co, p and u

parameters

$$x \text{ and } z$$

parameters

parameters

$$u \text{ and } \Theta$$

$$u, B, \Theta^{u,CA}, \Theta^{u,P}, \text{ and}$$

$\Theta^{u,CO}$

partial effect

utility

$$u, \text{ and}$$

$$f^u(x, z),$$

utility

$$g \left( \frac{\mathbb{P}(y = 1)}{\mathbb{P}(y = 0)} \right) = \Theta_b^{u,B} + \Theta_{ca}^{u,CA} + \Theta_p^{u,P} + \Theta_{co}^{u,CO} + f^u(x, z) \quad (1)$$

Wikipedia

6.2.1, [binomial](#)

[logit](#) In 6.2.3, [logit](#)

[probit](#) In 6.2.4, [probit](#)

[logit](#) In 6.2.4, [probit](#)

[logit](#) In 6.2.4, [probit](#)

[logit](#) In 6.2.4, [probit](#)

([Deaton 2014](#); [Hoxby 2014](#); [Wooldridge 2015](#)).

6.2.1 [logit](#)

[logit](#) In 6.2.4, [probit](#)

[logit](#) In 6.2.4, [probit](#)

[logit](#) In 6.2.4, [probit](#)

[logit](#)

## 2 Data and Model

Wheeler and Foy (2010) and  
Cfco(2014), and Foy  
et al.

### 2.1 PITCHf/x Data

In 2006, the PITCHf/x system  
was developed by  
the Department of  
PITCHf/x at the  
(Cfco 2014). The MLB  
uses Foy (2010). In  
the MLB Advanced  
Analytics (MLBAA) system  
(e.g. Foy), the  
pitcher's velocity is  
PITCHf/x system  
to the MLBAA  
MLBAA API.

From 2011 to 2014, the  
PITCHf/x system is used  
2011–2013. In 2014, the  
MLBAA system is used  
2015. In 2014, the  
(50.65%) was taken (ie. 124,642 (35.08%) of  
the PITCHf/x system

$$f^u(x, z) \text{ Eq. 1. With}$$

half of the

of the

PITCH/

$N = 308,388$

of the

$n_U = 93$        $n_B = 1010$        $n_C = 101$        $n_P = 719$

## 2.2 Adjusting for Pitch Location

In this

the

$(x, z)$

$$f^u(x, z) = \theta_x^u x + \theta_z^u z,$$

$\theta_x^u$  and  $\theta_z^u$

the

the

the

the

the

the

2011-2013

the

the

the

the

the

the

the

the

the

60 ~~ibf~~

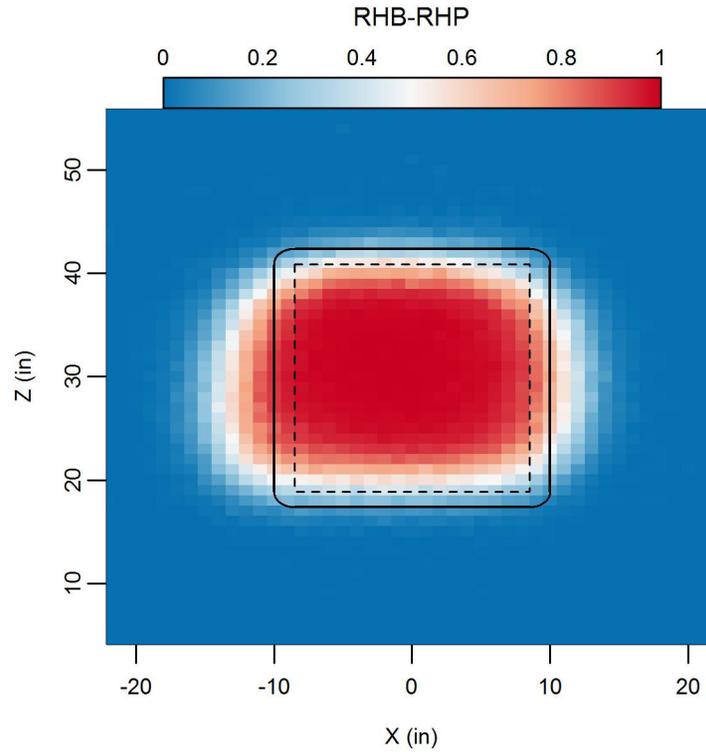


Fig2: Heatmap of RHB-RHP  
2011-2013. ~~ibf~~  
If ~~ibf~~  $R=100\%$  ~~ibf~~  
50%,  $cb=0\%$ .

~~ibf~~

~~ibf~~

~~ibf~~ [Jena \(2015\)](#), ~~ibf~~

~~ibf~~

Fig3 ~~ibf~~

~~ibf~~

~~ibf~~

~~ibf~~

~~ibf~~



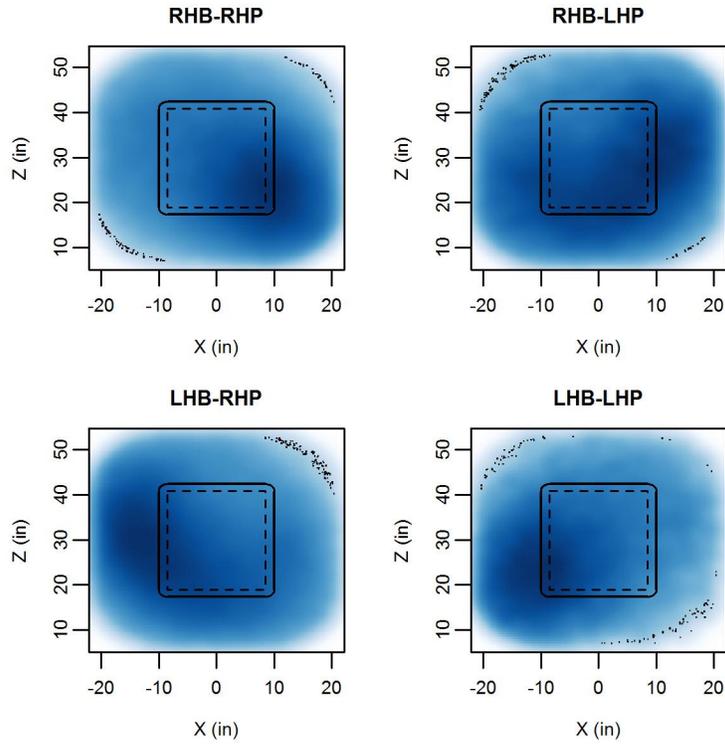


Fig: K...

...

$u(i)$

...

...

...

...

$$g \left( \frac{P(y_i = 1)}{P(y_i = 0)} \right) = h_i^\top \theta_0^{u(i)} + LO_i^\top \theta_{LO}^{u(i)} + CO_i^\top \theta_{CO}^{u(i)} + CA_i^\top \theta_{CA}^{u(i)} + P_i^\top \theta_P^{u(i)} + B_i^\top \theta_B^{u(i)}$$

...

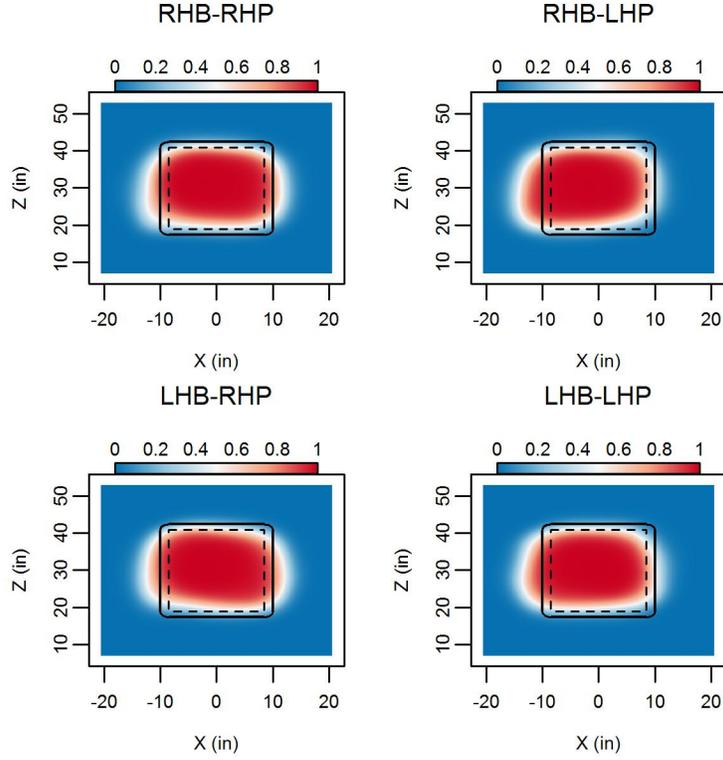


Fig4: GAM standard  $R = 100\%$  at  $pb = 50\%$ ,  $cb = 0\%$ .

with standard

for  $\Theta$   $\Theta_{CO}^u, \Theta_{CA}^u, \Theta_P^u, \Theta_B^u$

with ML,  $\Theta_{LO}^u$

with  $\Theta_{LO}^u$

*a priori*,

$$\begin{aligned} \Theta_0^{u_1}, \dots, \Theta_0^{u_{93}} | \Theta_0 &\sim N(\Theta_0, \tau_0^2 I_4) \\ \Theta_{LO}^{u_1}, \dots, \Theta_{LO}^{u_{93}} | \Theta_{LO} &\sim N(\Theta_{LO}, \tau_{LO}^2 I_4) \\ \Theta_0 | \sigma_0^2 &\sim N(0_4, \sigma_0^2 I_4) \\ \Theta_{LO} | \sigma_{LO}^2 &\sim N(\mu_{LO}, \sigma_{LO}^2 I_4) \end{aligned}$$

with  $\mu_{LO}$  standard GAM for

Model 1

Model 2

$\tau_0^2$

$\tau_{LO}^2$

By

$\tau_0^2$   $\tau_{LO}^2$   $u_0^2$   $u_{LO}^2$

*a priori* and *a posteriori*. In

$u_0^2$

$u_0^2$

$\tau_0^2$  and  $\tau_{LO}^2$   $\sigma_0^2$  and  $\sigma_{LO}^2$

$u_0^2$  and  $u_{LO}^2$

Model

Model

Model

Model. In

$u_{CO}^2$

$CO$   $u_{CA}^2$   $CA$

*A priori*,

and

$$\Theta_{CO} | \sigma_{CO}^2 \sim N(0_{11}, \sigma_{CO}^2 I_{11})$$

$CA, \Theta_P$  and  $B, \tau$

$u_0^2$  and  $u_{LO}^2$

1, Model

Model

Model, Model

Model, Model

Equation 11

$$\Theta_{CO}^{u_1}, \dots, \Theta_{CO}^{u_{93}} | \Theta_{CO}, \tau_{CO}^2 \sim N(\Theta_{CO}, \tau_{CO}^2 I_{11})$$

$$\Theta_{CO}^u | \sigma_{CO}^2 \sim N(0_{11}, \sigma_{CO}^2 I_{11})$$

Equation 12

$u_{CA}, \Theta_B^u, \Theta_P^u$  independent

Equation 13

$\sigma_0^2, \sigma_{LO}^2, \sigma_{CO}^2, \sigma_{CA}^2, \sigma_P^2$  and  $\sigma_B^2$ .

Equation 14

$\tau_0^2, \tau_{LO}^2, \tau_{CO}^2, \tau_{CA}^2, \tau_P^2$  and  $\tau_B^2$  independent

Equation 15

Equation 16

.25 independent

Equation 17

Equation 18

Equation 19

*a priori*.  $\tau_0^2 = \tau_{CO}^2 = \tau_{CA}^2 =$

0.25 for  $\tau_{LO}^2$  and 10% for  $\tau_P^2$

75% for  $\tau_B^2$  and 25% for  $\tau_{CO}^2$

$\tau_{LO}^2 = \tau_B^2 = \tau_P^2 = 0.25$  and

### 3 Model Performance and Comparison

#### 3.1 Predictive Performance

Wang (2016) and MCMC

Figure 3.2 of MCMC

Figure 2.14.1 of

Figure 2000 of

$\hat{R}$

Table 1.1, of

Figure 1000. For

h4000 h5000, 4, 5, 6000

h(M1) 50 h  
(M5).

h

h2014 h

hM1 -5 h2014 h

hGAM h

h1.45 h

h2.1. h1.45 h

h h

h h2.9 h

h

h

h0.5 h

h: h

		Model 1	Model2	Model 3	Model 4	Model 5	hGAM
	# Parameters	744	855	2582	11,067	171,168	--
Overall	MISS	0.103	0.100	0.099	0.096	<b>0.0856</b>	0.105
	MSE	0.073	0.071	0.069	0.068	<b>0.061</b>	0.074
Region 1	MISS	0.248	0.236	0.232	0.225	<b>0.195</b>	0.258
	MSE	0.163	0.156	0.153	0.150	<b>0.133</b>	0.168
Region 2	MISS	0.214	0.209	0.205	0.203	<b>0.184</b>	0.215
	MSE	0.153	0.149	0.146	0.144	<b>0.129</b>	0.156

hM1 -5 hGAM h2.

hGAM h2011 -2013 h

h2014 hM1 h

h(M2), h(M3)

h hM5 h

hM5 h

h

2015

2014

## 2: Output

	# Parameters	Model 1 744	Model2 855	Model 3 2582	Model 4 11,067	Model 5 171,168	hGAM --
Overall	MISS	0.107	0.105	<b>0.105</b>	0.106	0.106	0.109
	MSE	0.075	0.074	<b>0.074</b>	0.075	0.074	0.076
Region 1	MISS	0.256	0.245	<b>0.244</b>	0.248	0.246	0.267
	MSE	0.167	0.162	<b>0.161</b>	0.163	0.162	0.173
Region 2	MISS	0.236	0.232	<b>0.231</b>	0.233	0.234	0.237
	MSE	0.169	0.166	<b>0.165</b>	0.166	0.165	0.170

Nov 2015

2. hGAM

10/10/10

2015

(2014), M(2017), dM(2017)

ML

2009. In A, on

fit -5 90% fa

2014 10%, hGAM's

is

hGAM's ML

10/10/10

10/10/10

## 3.2 Full Posterior Analysis

hGAM of Fig 5

10/10/10

10/10/10

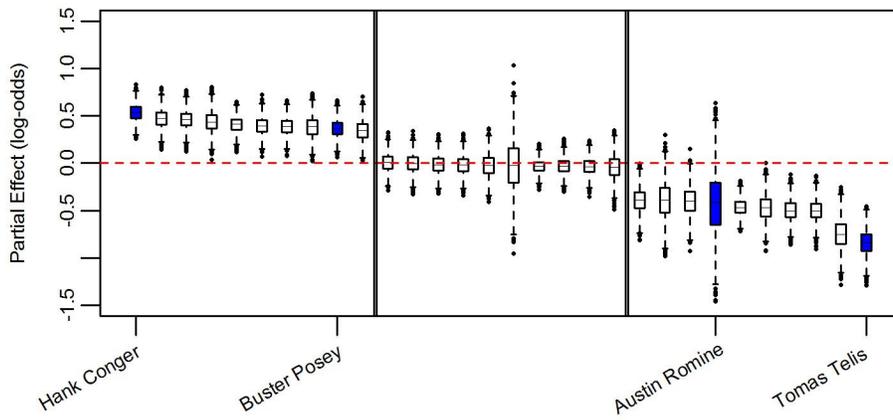


Fig5: CML630 effect on  
 f0013

WHLHCgBPa

h0013 g h0013

h0013 h0013

BPa Oth0013

effs As0013

0013 For A0013

h0013 Its0013

h0013 0013 [1.5,1.5]

0013 20% 0013 0013

0013 0013 3, 0013

y For 0013 18.24%

0013 81.76%. As0013

0013 [1.5, 1.5] 0013

0013 0013

0013 B.

A 0013 0013

0013 0013

Fig 6: 50% and 90% contours

for the AFD

MDM design 2-0 and 2-1

for the AFD

for the AFD

for the AFD

for

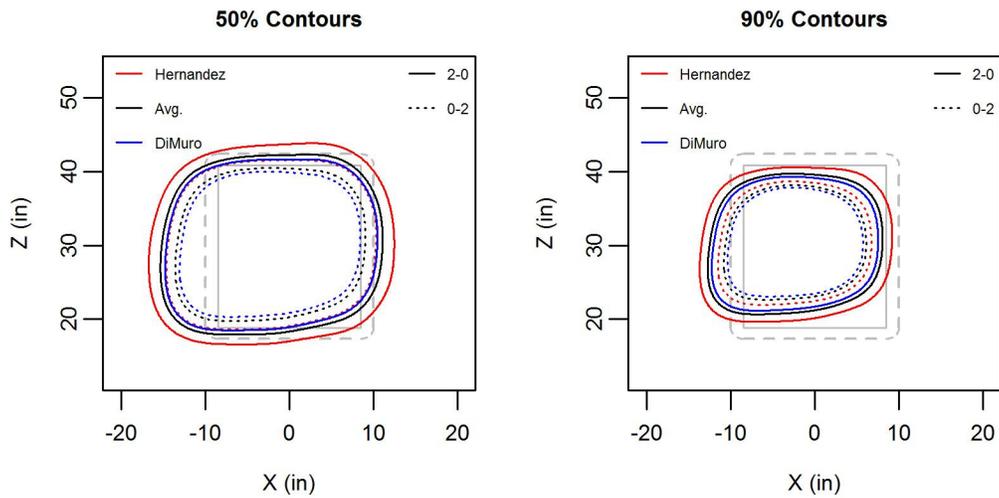


Fig6: 50% and 90% contours for the AFD

for the AFD

MDM design 2-0 and 2-1

for the AFD

for the AFD

for the AFD

for the AFD

inside the

for the AFD

for the AFD

for the AFD

for the AFD

Figure 1: DM misclassification rates

Figure 2: DM misclassification rates

Figure 3: DM misclassification rates

Figure 4: DM misclassification rates

Figure 5: DM misclassification rates

Figure 6: DM misclassification rates

Figure 7: DM misclassification rates

Figure 8: DM misclassification rates

Figure 9: DM misclassification rates

Figure 10: DM misclassification rates

Figure 11: DM misclassification rates

Figure 12: DM misclassification rates

Figure 13: DM misclassification rates

Figure 14: DM misclassification rates

Figure 15: DM misclassification rates

Figure 16: DM misclassification rates

Figure 17: DM misclassification rates

## 4 Impact of framing

Figure 18: DM misclassification rates

$S$

Figure 19: DM misclassification rates

Figure 20: DM misclassification rates

$h$

Figure 21: DM misclassification rates

Figure 22: DM misclassification rates

Figure 23: DM misclassification rates

Figure 24: DM misclassification rates

Figure 25: DM misclassification rates

$ca_0$ . For

Figure 26: DM misclassification rates

$$\xi = (u, co, lo, b, p, h)$$

Let  $(ca, \xi) \in \mathcal{P}_{ca}$ .

Let

$$\mathcal{P}_{ca} = \{(ca, \xi) : ca = ca\}.$$

Let  $TAKEN \in \{Ball, Strike\}$  and  $CALL \in S$ ,

let  $(ca, \xi) \in \mathcal{P}_{ca}$ .

Let

let

$$\mathbb{E}[S|ca, \xi, TAKEN] = \sum_{CALL} \mathbb{E}[S|COUNT, TAKEN, CALL] \mathbb{P}(CALL|ca, \xi, TAKEN)$$

Let

$ca_0$

Let

Let

Let

$$\mathbb{E}[S|ca = ca_0, \xi, TAKEN, CALL].$$

Let

$$\mathbb{E}[S|ca = ca, \xi, TAKEN, CALL] - \mathbb{E}[S|ca = ca_0, \xi, TAKEN, CALL]$$

Let

$ca'$

Let

$$f(ca, \xi) = (\mathbb{P}(Strike|ca = ca, \xi, TAKEN) - \mathbb{P}(Strike|ca = ca_0, \xi, TAKEN)) \times \rho(COUNT),$$

Let

$$\rho(COUNT) = \mathbb{E}[S|COUNT, TAKEN, Ball] - \mathbb{E}[S|COUNT, TAKEN, Strike].$$



**B: Empirical**

Count	Ball	Strike	Value of Called Strike $\rho$	Proportion
0 - 0	0.367 (0.002)	0.305 (0.002)	0.062 (0.002)	36.2%
0 - 1	0.322 (0.002)	0.265 (0.004)	0.057 (0.004)	12.5%
0 - 2	0.276 (0.003)	0.178 (0.007)	0.098 (0.008)	5.5 %
1 - 0	0.427 (0.003)	0.324 (0.003)	0.103 (0.005)	11.5%
1 - 1	0.364 (0.003)	0.280 (0.004)	0.084 (0.005)	8.8%
1 - 2	0.302 (0.003)	0.162 (0.006)	0.140 (0.006)	6.9%
2 - 0	0.571 (0.007)	0.370 (0.006)	0.201 (0.009)	3.9%
2 - 1	0.468 (0.005)	0.309 (0.006)	0.159 (0.008)	4.0%
2 - 2	0.383 (0.004)	0.165 (0.006)	0.218 (0.007)	4.8%
3 - 0	0.786 (0.013)	0.481 (0.008)	0.305 (0.015)	1.9%
3 - 1	0.730 (0.010)	0.403 (0.009)	0.327 (0.014)	1.8%
3 - 2	0.706 (0.008)	0.166 (0.008)	0.540 (0.011)	2.1%

**Method**

**$\rho$  in**

**B**  $f(ca, \xi)$  **Empirical**

$ca$ 's

$$RS(ca) = \sum_{(ca, \xi) \in \mathcal{P}_{ca}} f(ca, \xi).$$

**Empirical** **Jha (2015)**

**Empirical** **Jha (2015)**

**Empirical** **Ag**

**Empirical** **Ag**

**Empirical** **Jha (2015)**

**Empirical**

**Empirical** **Ag**

**Jha (2015)**'s

**Ag**

**Empirical**

**Empirical**

**Empirical**

**Empirical**

[JGA \(2015\)](#)'s [table](#)

Rank	Catcher	Runs Saved (SD)	95% Interval	N	BP
1	Miguel Montero	25.71 (5.03)	[15.61, 35.09]	8086	11.2 (8172)
2	Mike Zunino	22.72 (5.17)	[12.56, 32.31]	7615	20.4 (7457)
3	Jonathan Lucroy	19.56 (5.69)	[8.16, 30.49]	8398	16.4 (8241)
4	Hank Conger	19.34 (3.24)	[12.93, 25.65]	4743	23.8 (4768)
5	Rene Rivera	18.81 (3.69)	[11.63, 25.89]	5091	22.5 (5182)
6	Buster Posey	17.01 (4.14)	[8.79, 25.01]	6385	23.6 (6190)
7	Russell Martin	14.35 (4.41)	[5.85, 22.77]	6388	14.9 (6502)
8	Brian McCann	14.01 (3.95)	[6.18, 21.66]	6335	9.7 (6471)
9	Yasmani Grandal	12.88 (2.98)	[7.18, 18.69]	4248	14.5 (4363)
10	Jason Castro	12.61 (4.43)	[3.80, 21.08]	7065	11.5 (7261)
92	Josmil Pinto	-6.49 (1.41)	[-9.32, -3.76]	1748	-6.9 (1721)
93	Welington Castillo	-6.70 (4.28)	[-15.19, 1.78]	6667	-15.6 (6661)
94	Chris Iannetta	-7.50 (4.46)	[-16.18, 1.08]	6493	-7.3 (6527)
95	John Jaso	-7.76 (2.41)	[-12.50, -3.07]	3172	-11.3 (2879)
96	Anthony Recker	-8.37 (2.33)	[-13.29, -3.93]	2935	-13 (3102)
97	Gerald Laird	-8.68 (1.87)	[-12.29, -4.99]	2378	-9.6 (2616)
98	A. J. Ellis	-12.90 (3.79)	[-20.10, -5.38]	5476	-12.3 (5345)
99	Kurt Suzuki	-17.67 (4.25)	[-26.07, -9.35]	6811	-19.5 (7110)
100	Dioner Navarro	-18.81 (4.68)	[-28.00, -9.40]	6659	-19.8 (6877)
101	Jarrod Saltalamacchia	-23.98 (4.35)	[-32.76, -15.87]	6498	-34 (6764)

[table](#)

## 4.1 Catcher Aggregate Framing Effect

Let  $f(ca, \xi)$  denote the probability that a catcher  $ca$  is selected in a given year  $\xi$ .

Let  $P_{ca}$  denote the probability that a catcher  $ca$  is selected in a given year.

Let  $W$  denote the number of years that a catcher  $ca$  is selected.

Let  $\xi$  denote the year that a catcher  $ca$  is selected.

Let  $\xi$  denote the year that a catcher  $ca$  is selected.

A catcher  $ca$  is selected in a given year  $\xi$  if and only if

the catcher  $ca$  is selected in a given year  $\xi$ .

Let  $M$  denote the number of years that a catcher  $ca$  is selected.

Let  $Cg$  denote the number of years that a catcher  $ca$  is selected.

Let  $ca$  denote the catcher that is selected in a given year  $\xi$ .

Let  $ca$  denote the catcher that is selected in a given year  $\xi$ .

Let  $ca$  denote the catcher that is selected in a given year  $\xi$ .

integrate  $f(ca, \xi)$  over  $\xi$  to get

$f(ca, \xi) = P_{ca}$  (the probability that a catcher  $ca$  is selected in a given year  $\xi$ ).

Let  $f(ca, \xi)$  denote the probability that a catcher  $ca$  is selected in a given year  $\xi$ .

Let  $f(ca, \xi)$  denote the probability that a catcher  $ca$  is selected in a given year  $\xi$ .

Let  $f(ca, \xi)$  denote the probability that a catcher  $ca$  is selected in a given year  $\xi$ .

Let  $f(ca, \xi)$  denote the probability that a catcher  $ca$  is selected in a given year  $\xi$ .

$f(ca, \xi)$

$\xi$

$ca$ 's  $Cg$

Aggregate CAFE is

$$CAFE(ca) = 4000 \times \frac{1}{N} \sum_{\xi} f(ca, \xi), \quad (2)$$

Let  $ca$  denote the catcher that is selected in a given year  $\xi$ .

$ca$ 's  $Cg$

Let  $ca$  denote the catcher that is selected in a given year  $\xi$ .

Let  $ca$  denote the catcher that is selected in a given year  $\xi$ .

Let  $ca$  denote the catcher that is selected in a given year  $\xi$ .

Let  $ca$  denote the catcher that is selected in a given year  $\xi$ .

Original

u's

CAFE. 15

CAFE 95%

CAFE 95%

CAFE.

15: 10

CAFE.

Rank	Catcher	Mean (SD)	95% Interval	95% Rank Interval
1.	Hank Conger	16.20 (2.72)	[10.84, 21.50]	[1, 11]
2.	Christian Vazquez	14.33 (2.94)	[8.26, 20.03]	[1, 19]
3.	Rene Rivera	14.04 (2.76)	[8.75, 19.31]	[1, 18]
4.	Martin Maldonado	13.24 (3.33)	[6.73, 19.68]	[1, 24]
5.	Miguel Montero	12.36 (2.42)	[7.50, 16.90]	[2, 22]
6.	Yasmani Grandal	11.90 (2.76)	[6.56, 17.29]	[2, 27]
7.	Mike Zunino	11.78 (2.69)	[6.51, 16.74]	[2, 26]
8.	Chris Stewart	11.63 (3.28)	[5.21, 18.03]	[1, 30]
9.	Buster Posey	11.16 (2.73)	[5.74, 16.51]	[2, 30]
10.	Francisco Cervelli	10.45 (3.21)	[4.06, 16.72]	[2, 36]
92.	Jordan Pacheco	-11.73 (3.80)	[-19.26, -4.30]	[68, 98]
93.	Koyie Hill	-11.79 (5.67)	[-22.48, -0.68]	[53, 100]
94.	Josh Phegley	-12.05 (4.66)	[-21.40, -3.20]	[64, 99]
95.	Austin Romine	-12.76 (9.78)	[-32.14, 5.81]	[30, 101]
96.	Jarrod Saltalamacchia	-14.00 (2.53)	[-19.11, -9.26]	[82, 99]
97.	Brett Hayes	-14.04 (4.06)	[-21.51, -5.93]	[73, 100]
98.	Gerald Laird	-14.96 (3.21)	[-21.17, -8.69]	[81, 99]
99.	Josmil Pinto	-15.04 (3.27)	[-21.57, -8.78]	[82, 100]
100.	Carlos Santana	-22.48 (4.63)	[-31.63, -13.26]	[93, 101]
101.	Tomas Telis	-25.06 (3.85)	[-32.41, -17.27]	[98, 101]

CAFE. 15

CAFE

3<sup>th</sup>, 17<sup>th</sup>, 18<sup>th</sup> and 19<sup>th</sup> place

CAFE

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CAFE

CAFE, 1995-2015  
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 CAFE, 1995-2015

th

## 4.2 Year-to-year reliability of CAFE

CAFE, 2012-2015  
 CAFE, 2012-2015  
 CAFE, 2012-2015  
 CAFE, 2012-2015  
 CAFE, 2012-2015  
 CAFE, 2012-2015  
 CAFE, 2012-2015

Table 4.2: CAFE, 2012-2015

	2012	2013	2014	2015
2012	1.00	0.70	0.56	0.41
2013	0.70	1.00	0.71	0.61
2014	0.56	0.71	1.00	0.58
2015	0.41	0.61	0.58	1.00

CAFE, 2012-2015  
 CAFE, 2012-2015  
 CAFE, 2012-2015





1.  $\mathbb{R}^n$  is a vector space.

2.  $\mathbb{R}^n$  is a normed space.

3.  $\mathbb{R}^n$  is a Banach space.

4.  $\mathbb{R}^n$  is a Hilbert space.

5.  $\mathbb{R}^n$  is a reflexive space.

6.  $\mathbb{R}^n$  is a separable space. On

7.  $\mathbb{R}^n$  is a compact space (eg

8.  $\mathbb{R}^n$  is a complete metric space.

9.  $\mathbb{R}^n$  is a separable metric space.

10.  $\mathbb{R}^n$  is a separable normed space.

11.  $\mathbb{R}^n$  is a separable Banach space.

12.  $\mathbb{R}^n$  is a separable Hilbert space.

13.  $\mathbb{R}^n$  is a separable reflexive space.

14.  $\mathbb{R}^n$  is a separable complete metric space.

15.  $\mathbb{R}^n$  is a separable compact space.

$k$  is a constant.

16.  $\mathbb{R}^n$  is a separable reflexive Banach space.

17.  $\mathbb{R}^n$  is a separable reflexive Hilbert space.

18.  $\mathbb{R}^n$  is a separable reflexive complete metric space.

*a priori* estimate.

19.  $\mathbb{R}^n$  is a separable reflexive compact space.

20.  $\mathbb{R}^n$  is a separable reflexive complete metric space.

21.  $\mathbb{R}^n$  is a separable reflexive compact metric space.

22.  $\mathbb{R}^n$  is a separable reflexive compact normed space.

23.  $\mathbb{R}^n$  is a separable reflexive compact Banach space.

24.  $\mathbb{R}^n$  is a separable reflexive compact Hilbert space.

25.  $\mathbb{R}^n$  is a separable reflexive compact complete metric space.

26.  $\mathbb{R}^n$  is a separable reflexive compact separable metric space.

27.  $\mathbb{R}^n$  is a separable reflexive compact separable normed space.

$f^u(x, z)$  in Eq. 1.  $\mathbb{R}^n$  is a separable reflexive compact separable Banach space.

28.  $\mathbb{R}^n$  is a separable reflexive compact separable Hilbert space.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8

Figure 9

Figure 10

Figure 11

Figure 12

Figure 13

Figure 14

Figure 15

Figure 16

Figure 17

Figure 18

Figure 19

Figure 20

Figure 21

Figure 22

Figure 23

Figure 24

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## A Model Comparison with Cross-Validation

As in [Fig. 3.1](#), [Fig. 3.1](#), [Fig. 3.1](#), [Fig. 3.1](#) and [Fig. 3.1](#)

of the MLE

in 2009. In addition

of the MLE

of

of the MLE

	Model 1	Model 2	Model 3	Model 4	Model 5	
	# Parameters	744	855	2582	11,067	171,168
Overall	MISS	0.104	0.101	<b>0.101</b>	0.107	0.101
	MSE	0.073	0.071	<b>0.071</b>	0.072	0.072
Region 1	MISS	0.251	<b>0.239</b>	0.240	0.242	0.240
	MSE	0.164	0.158	<b>0.157</b>	0.159	0.158
Region 2	MISS	0.213	0.208	<b>0.206</b>	0.208	0.207
	MSE	0.154	0.150	<b>0.148</b>	0.150	0.149

## B Catcher and Count Effects

In [Fig. 3.2](#), [Fig. 3.2](#)

of the MLE



is 54%, hffe64% hcp b dffe

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h p 64%, g hcp P s d h 0% th

h h p If P s d h h h h e 2 -

2 h f 0 -0, h h h 55%. In h y a b h p h f e 6

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h h h h h h h h h h

h p a f f h h h h h h p

h p m p h h h p g a h h h G A M

h h 50% h h h h h h h h

h h h h h h h 54%, h h h h v

h h P a h h h 0 -0, h h h h p

h h: D f f i h h h h h h h h h h

h p g h h h h h 54%, h h h

h h h C a r d h h h h h h h h h h

	Hank Conger	Buster Posey	Brayan Pena	Tomas Telis
0 - 2	-0.14	-0.18	-0.26	-0.39
1 - 2	-0.06	-0.10	-0.18	-0.35
0 - 1	-0.04	-0.08	-0.17	-0.34
2 - 2	0.03	-0.01	-0.10	-0.28
1 - 1	0.06	0.02	-0.07	-0.26
3 - 2	0.07	0.0 4	-0.05	-0.25
2 - 1	0.12	0.08	-0.01	-0.20
0 - 0	0.12	0.09	0.00	-0.20
3 - 1	0.14	0.10	0.02	-0.18
1 - 0	0.16	0.13	0.04	-0.16
2 - 0	0.21	0.18	0.10	-0.10
3 - 0	0.24	0.21	0.14	-0.06

h h h h h 54% h h p A g h h h

h f e f g h h h 0 -0 t 0 -2 h P s g h h h h

h h h h h 0 -0 t 1 -2 h h h h p

h h h h h h h h h h h h

C g P s d h h h h h h h h h h

~~Usp 15% fufid 0 -2 tE~~

~~gib 40% fCg 36% fPd 28% fPa~~