Kelly Criterion for 2021 Elite 8

this was wrong initially in my dataset, sorry...

	team	opp	teamWon	p	alpha	ml	implied.p	plusEV
1	2 Houston	12 Oregon St	1	0.7553	1.277778	-360	0.7826087	0
2	12 Oregon St	2 Houston	0	0.2447	3.600000	260	0.2777778	0
3	1 Baylor	3 Arkansas	1	0.7737	1.327869	-305	0.7530864	1
4	3 Arkansas	1 Baylor	0	0.2263	3.450000	245	0.2898551	0
5	1 Gonzaga	6 USC	1	0.7726	1.229885	-435	0.8130841	0
6	6 USC	1 Gonzaga	0	0.2274	4.250000	325	0.2352941	0
7	1 Michigan	11 UCLA	0	0.6486	1.289855	-345	0.7752809	0
8	11 UCLA	1 Michigan	1	0.3514	3.750000	275	0.2666667	1

p = win probability, from 538 "true probs"

mL = moneyline odds, from dRaftKings

Implied.p = the win probability that is implied from the moneyline odds

Ex $mL = -360 \Rightarrow \text{Implied.p} = \frac{360}{360+100} = .7826$

$mL = +260 \Rightarrow \text{Implied.p} = \frac{100}{260+100} = .2778$

$$\text{plusEV} = \begin{cases} \text{true if} & p > \text{implied.p} \\ \text{false if} & p \le \text{implied.p} \end{cases}$$

"positive expected value bet"

- One betting strategy is to only make +EV bets. But this is not necessarily Kelly optimal... see the end of these notes

α_s the odds paid on the occurrence of the s'th transmitted symbol, i.e., α_s is the number of dollars returned for a one-dollar bet (including that one dollar). $s \in \{1, 2\}$

- Need to convert moneyline odds to α.

Ex $mL = -360 \Rightarrow$ bet \$360 to profit \$100

$\Rightarrow$ bet \$1 to profit $\frac{100}{360}$

$\Rightarrow \alpha = 1 + \frac{100}{360} = 1.2778$

$mL = +260 \Rightarrow$ bet \$100 to profit 260

$\Rightarrow$ bet \$1 to profit $\frac{260}{100}$

$\Rightarrow \alpha = 1 + \frac{260}{100} = 3.6$

	team	opp	teamWon	p	alpha	ml	implied.p	plusEV
1	2 Houston	12 Oregon St	1	0.7553	1.277778	-360	0.7826087	0
2	12 Oregon St	2 Houston	0	0.2447	3.600000	260	0.2777778	0
3	1 Baylor	3 Arkansas	1	0.7737	1.327869	-305	0.7530864	1
4	3 Arkansas	1 Baylor	0	0.2263	3.450000	245	0.2898551	0
5	1 Gonzaga	6 USC	1	0.7726	1.229885	-435	0.8130841	0
6	6 USC	1 Gonzaga	0	0.2274	4.250000	325	0.2352941	0
7	1 Michigan	11 UCLA	0	0.6486	1.289855	-345	0.7752809	0
8	11 UCLA	1 Michigan	1	0.3514	3.750000	275	0.2666667	1

Kelly Step 1

For each game,

(a) Permute indices so that $p(s)\alpha_s \geq p(s+1)\alpha_{s+1}$

	team	opp	teamWon	p	alpha	ml	implied.p	plusEV	p.times.alpha
1	2 Houston	12 Oregon St	1	0.7553	1.277778	-360	0.7826087	0	0.9651056
2	12 Oregon St	2 Houston	0	0.2447	3.600000	260	0.2777778	0	0.8809200
3	1 Baylor	3 Arkansas	1	0.7737	1.327869	-305	0.7530864	1	1.0273721
4	3 Arkansas	1 Baylor	0	0.2263	3.450000	245	0.2898551	0	0.7807350
5	6 USC	1 Gonzaga	0	0.2274	4.250000	325	0.2352941	0	0.9664500
6	1 Gonzaga	6 USC	1	0.7726	1.229885	-435	0.8130841	0	0.9502092
7	11 UCLA	1 Michigan	1	0.3514	3.750000	275	0.2666667	1	1.3177500
8	1 Michigan	11 UCLA	0	0.6486	1.289855	-345	0.7752809	0	0.8366000

4 games

For each game,

b = the fraction Not bet on this game.

Kelly step 2

(b) Set b equal to the minimum positive value of

$$F_t = \frac{1-p_t}{1-\sigma_t} \quad \text{where} \quad p_t = \sum_1^t p(s), \quad \sigma_t = \sum_1^t \frac{1}{\alpha_s}$$

$t \in \{0,1\}$ is the index that minimizes F_t. $F_0 = 1$.

- $t=0$ if do not bet on this game $\iff p(1)\alpha_1 < 1$ (both bets are negative EV!)
- $t=1$ if bet on the team with the higher $p(s)\alpha_s$ value $\iff p(1)\alpha_1 > 1$
- $t \neq 2$ since can't bet on team with lowest $p(s)\alpha_s$ value

Consequence for 2-team Kelly, bet on a team $\Rightarrow$ plus EV bet!

	team	opp	teamWon	p	alpha	ml	implied.p	plusEV	p.times.alpha	F_0	F_1	t	b
1	2 Houston	12 Oregon St	1	0.7553	1.277778	-360	0.7826087	0	0.9651056	1	1.1256200	0	1.0000000
2	12 Oregon St	2 Houston	0	0.2447	3.600000	260	0.2777778	0	0.8809200	NA	NA	NA	1.0000000
3	1 Baylor	3 Arkansas	1	0.7737	1.327869	-305	0.7530864	1	1.0273721	1	0.9165150	1	0.9165150
4	3 Arkansas	1 Baylor	0	0.2263	3.450000	245	0.2898551	0	0.7807350	NA	NA	NA	1.0000000
5	6 USC	1 Gonzaga	0	0.2274	4.250000	325	0.2352941	0	0.9664500	1	1.0103231	0	1.0000000
6	1 Gonzaga	6 USC	1	0.7726	1.229885	-435	0.8130841	0	0.9502092	NA	NA	NA	1.0000000
7	11 UCLA	1 Michigan	1	0.3514	3.750000	275	0.2666667	1	1.3177500	1	0.8844545	1	0.8844545
8	1 Michigan	11 UCLA	0	0.6486	1.289855	-345	0.7752809	0	0.8366000	NA	NA	NA	1.0000000

$$\begin{cases} F_0 = 1 \\ F_1 = \dfrac{1-p(1)}{1-1/\alpha(1)} = \dfrac{\alpha(1)-\alpha(1)p(1)}{\alpha(1)-1} \end{cases}$$

so to bet on team 1, we need $F_1 < F_0 = 1$, so need $\alpha(1)p(1) > 1$.

Kelly Step 3

(c) Set $a(s) = p(s) - b/\alpha_s$ or zero, whichever is larger. (The $a(s)$ will sum to $1 - b$.)

Here: $a = 1-b$, since 2 outcomes per game

	team	opp	teamWon	p	alpha	ml	implied.p	plusEV	p.times.alpha	b	a	a_
1	2 Houston	12 Oregon St	1	0.7553	1.277778	-360	0.7826087	0	0.9651056	1.0000000	0.0000000	0.00000000
2	12 Oregon St	2 Houston	0	0.2447	3.600000	260	0.2777778	0	0.8809200	1.0000000	0.0000000	0.00000000
3	1 Baylor	3 Arkansas	1	0.7737	1.327869	-305	0.7530864	1	1.0273721	0.9165150	0.0834850	0.04174250
4	3 Arkansas	1 Baylor	0	0.2263	3.450000	245	0.2898551	0	0.7807350	1.0000000	0.0000000	0.00000000
5	6 USC	1 Gonzaga	0	0.2274	4.250000	325	0.2352941	0	0.9664500	1.0000000	0.0000000	0.00000000
6	1 Gonzaga	6 USC	1	0.7726	1.229885	-435	0.8130841	0	0.9502092	1.0000000	0.0000000	0.00000000
7	11 UCLA	1 Michigan	1	0.3514	3.750000	275	0.2666667	1	1.3177500	0.8844545	0.1155455	0.05777273
8	1 Michigan	11 UCLA	0	0.6486	1.289855	-345	0.7752809	0	0.8366000	1.0000000	0.0000000	0.00000000

Step 4: Make Bets, and see how we did!

we only make +EV bets! This is because it's 2-team Kelly (see later)

$B = \$1000$, total bet size

	team	opp	teamWon	p	alpha	ml	implied.p	plusEV	p.times.alpha	a_	bet	profit
1	2 Houston	12 Oregon St	1	0.7553	1.277778	-360	0.7826087	0	0.9651056	0.00000000	0.00000	0.00000
2	12 Oregon St	2 Houston	0	0.2447	3.600000	260	0.2777778	0	0.8809200	0.00000000	0.00000	0.00000
3	1 Baylor	3 Arkansas	1	0.7737	1.327869	-305	0.7530864	1	1.0273721	0.04174250	41.74250	13.68607
4	3 Arkansas	1 Baylor	0	0.2263	3.450000	245	0.2898551	0	0.7807350	0.00000000	0.00000	0.00000
5	6 USC	1 Gonzaga	0	0.2274	4.250000	325	0.2352941	0	0.9664500	0.00000000	0.00000	0.00000
6	1 Gonzaga	6 USC	1	0.7726	1.229885	-435	0.8130841	0	0.9502092	0.00000000	0.00000	0.00000
7	11 UCLA	1 Michigan	1	0.3514	3.750000	275	0.2666667	1	1.3177500	0.05777273	57.77273	158.87500
8	1 Michigan	11 UCLA	0	0.6486	1.289855	-345	0.7752809	0	0.8366000	0.00000000	0.00000	0.00000

Total profit from Elite 8: $172.56

Cool! other Rounds didn't do as well, but still made money!

Kelly on Entire March Madness 2021

	Bankroll.Name	Bankroll	Gained
	<chr>	<dbl>	<dbl>
1	initial bankroll	1000	NA
2	post First 4	878.	0
3	post Round 1	887.	1
4	post Round 2	817.	0
5	post Sweet 16	843.	1
6	post Elite 8	989.	1
7	post Final 4	955.	0
8	final bankroll	1125.	1

- Start with $1000
- Roll over the bankroll round-to-round
- Profit! Took a beating in the 1st four games

	Bankroll.Name	Bankroll	Gained
	<chr>	<dbl>	<dbl>
1	initial bankroll	1000	NA
2	post Round 1	1011.	1
3	post Round 2	931.	0
4	post Sweet 16	961.	1
5	post Elite 8	1126.	1
6	post Final 4	1087.	0
7	final bankroll	1282.	1

- Remove the First 4 games, which we know are unpredictable crapshoots. Kelly does better!