

Example of a Kelly-Optimal, Negative EV Bet

3 Horses.

True win probabilities

$$\vec{p} = (p_1, p_2, p_3) = (.6, .3, .1)$$

Moneyline Odds

$$\vec{ML} = (+100, +200, +700)$$

α odds (from Kelly)

$$\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3) = (2, 3, 8)$$

$$\left\{ \begin{array}{l} p_1 \alpha_1 = 1.2 > 1 \Rightarrow \text{horse 1 is +EV} \\ p_2 \alpha_2 = .9 < 1 \Rightarrow \text{horse 2 is -EV} \\ p_3 \alpha_3 = .8 < 1 \Rightarrow \text{horse 3 is -EV} \\ p_1 \alpha_1 > p_2 \alpha_2 > p_3 \alpha_3 \Rightarrow \text{correct ordering} \end{array} \right.$$

$$0 < \underline{.6} = \frac{1 - p_1 - p_2}{1 - \frac{1}{\alpha_1} - \frac{1}{\alpha_2}} < .8 = \frac{1 - p_1}{1 - \frac{1}{\alpha_1}}$$

$$b = \underline{.6}$$

$$a_1 = \max\left(p_1 - \frac{b}{\alpha_1}, 0\right) = .3$$

$$a_2 = \max\left(p_2 - \frac{b}{\alpha_2}, 0\right) = .1$$

$$a_3 = \max\left(p_3 - \frac{b}{\alpha_3}, 0\right) = 0$$

make a negative EV bet on horse 2!